Vidyamandir Classes

Gurukul for IITJEE & Medical Preparation

SOLUTIONS

Joint Entrance Exam | IITJEE-2019

09th APRIL 2019 | Evening Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(1) Conservation of angular momentum

$$\left[\frac{ML^2}{12}+2m(0)^2\right]\omega_0\left[\frac{ML^2}{12}+2\times m\left(\frac{L}{2}\right)^2\right]\omega$$

$$\omega=\frac{M\omega_0}{M+6m}$$

2.(3) We have confidence only till two places of decimal in actual measurement so the final answer accurate to the just 2 decimal place.

Note : It is actual measurement which using some device with least count.

3.(3) $\rho=\frac{m}{ne^2\tau}$ $\quad =1.67\times10^{-8}\,\Omega m$

4.(2) $x(t)=at+bt^2-ct^3$, $\quad v=\frac{dx(t)}{dt}=a+2bt-3ct^2$

$$a=\frac{d^2x(t)}{dt^2}=2b-6ct\,,\quad a=0]$$

$$g=\frac{b}{3c},\qquad v=a+2b\left(\frac{b}{3c}\right)-3c\left(\frac{b}{3c}\right)^2$$

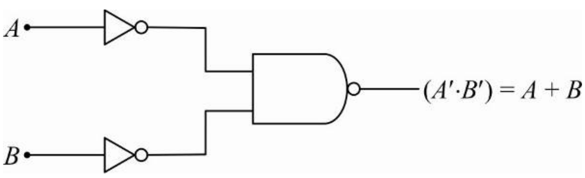
$$v=a+\frac{b^2}{3c}$$

5.(4) Truth table can be formed as

A	*B*	Equivalent
0	0	0
0	1	1
1	0	1
1	1	1

Hence the equivalent is "OR" gate

Method-2



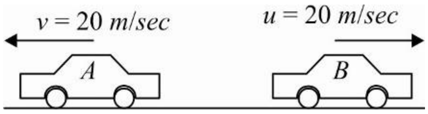
OR (GATE)

6.(4) Limit of resolution $=\frac{1.22\lambda}{d}$

$$=\frac{1.22\times600\times10^{-9}}{250\times10^{-2}}$$

$$=2.9\times10^{-7}\,rad$$

7.(3)



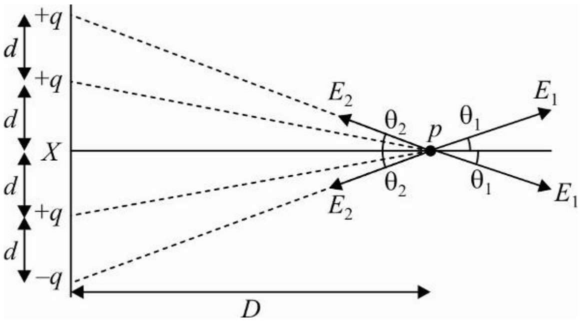
$$f = f_0\left[\frac{v - v_c}{v + v_c}\right]$$

When moving away from each other

$$2000 = f_0\left[\frac{340 - 20}{340 + 20}\right]$$

$$f_0 = 2250\,Hz$$

8.(1) Electric field at $p = 2E_1\cos\theta_1 - 2E_1\cos\theta_2$



$$= \frac{2Kq}{(d^2 + D^2)} \times \frac{D}{(d^2 + D^2)^{1/2}} - \frac{2Kq}{[(2d)^2 + D^2]} \times \frac{D}{[(2d)^2 + D^2]^{1/2}}$$

$$= 2KqD\left[(d^2 + D^2)^{-3/2} - (4d^2 + D^2)^{-3/2}\right]$$

$$= \frac{2KqD}{D^3}\left[\left(1 + \frac{d^2}{D^2}\right)^{-3/2} - \left(1 + \frac{4d^2}{D^2}\right)^{-3/2}\right]$$

Applying binomial approximation $\therefore$ $d << D$

$$= \frac{2KqD}{D^3}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3 \times 4d^2}{2D^2}\right)\right]$$

$$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$$

$$= \frac{9Kqd^2}{D^4}$$

9.(3) $T = MB$

$C\phi = NIAB$

$$B = \frac{10^{-6} \times \pi}{175 \times 10^{-4} \times 10^{-3}} = 10^{-3}T$$

10.(1) $\rho(r) = \frac{K}{r^2}$, $\quad \frac{Mv^2}{R} = \frac{GM'}{R^2}$

$$\frac{Mv^2}{R} = \frac{G\int_0^R \rho(r)4\pi r^2 dn}{R^2}, \qquad m\left(\frac{2\pi}{T}\right)^2 R = \frac{GK4\pi R}{R^2}$$

$$\frac{T}{R} = \text{constant}$$

11.(2) $P = \frac{I}{C}(a + 2r)$

$= \frac{I}{C} \times \left(\frac{2}{4} + \frac{3}{4}\right)$

$P = \frac{5}{4}\frac{I}{C} = 20 \cdot 8 \times 10^{-8}$

12.(2) $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

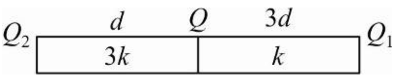
$\frac{\vec{dt^2}}{dt^2} = 30\hat{i} + (-40)\hat{j}$

$a = 50\ m/\sec^2$

13.(1) $L = 2.0\ m$ $\quad f = 240\ Hz$

$f = \frac{3v}{2L} = 240$, $\frac{3v}{4} = 240$

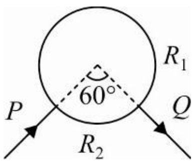
$v = 320$, $f_0 = \frac{v}{2L} = 80\,Hz$

14.(2) 

$d\frac{dQ}{dt} = \frac{3KA(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$

$\theta = \frac{\theta_1}{10} + \frac{9\theta_2}{10}$

15.(1) $R = 3\Omega$, $\quad R \propto l^2$

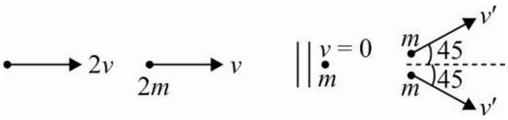


$R' = 2^2 \times 3 = 12\ \Omega$

$R_1 = \frac{5\pi}{6\pi} \times R' = 10\ \Omega$

$R_2 = \frac{\pi}{6\pi} \times R' = 2\ \Omega$

$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{5}{3}\Omega$

16.(1) 

Linear momentum conservation

$m2v + 2mv = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$, $\quad v' = 2\sqrt{2}v$ c

17.(2) $\phi_{\text{outer}}(\mu_0 nKte^{-\alpha t})4\pi R^2$, $\qquad \varepsilon = \frac{-d\phi}{dt} = Ce^{-\alpha t}[1 - \alpha t]\,b$

$i_{\text{induced}} = \frac{-Ce^{-\alpha t}[1 - at]}{(\text{Resistance})}$

At $t = 0$ $i_{\text{induced}} = -ve$

18.(4) Potential Energy of spring = Total Head Energy

$$\frac{1}{2}kA^2 = m_1 s_1 \Delta\theta + m_2 s_2 \Delta\theta$$

$$\Delta\theta = \frac{\frac{1}{2}kA^2}{m_1 s_1 + m_2 s_2}, \qquad \Delta\theta = \frac{400 \times \left(\frac{2}{100}\right)^2}{(4184 + 200)} = 3.6 \times 10^{-5} k$$

19.(3) $\alpha = 20\ rad/\sec^2$

$\omega = \omega_0 + \alpha t$

$\omega = 20t$

$\frac{1}{2}I\omega^2 = 1200$

$\frac{1}{2} \times 1.5 \times (20t)^2 = 1200$

$t = 2\ sec$

20.(4) $E_n = -13.6\frac{z^2}{n^2}$

$n = 2, \quad z = 2$

$E = -13.6 \times \frac{2^2}{2^2} = -13.6\ eV$

21.(3) Magnification is 2

If image is real, $x_1 = \frac{3f}{2}$

If image is virtual, $x_2 = \frac{f}{2}$

$\frac{x_1}{x_2} = 3:1$

22.(3) $\frac{1}{f_1} = \frac{1}{2} \times \frac{2}{18} = \frac{1}{18}$

$\frac{1}{f_2} - \frac{(\mu_1 - 1)}{-18}$

When μ_1 is filled between lens and mirror

$P = \frac{2}{18} - \frac{2}{18}(\mu_1 - 1) = \frac{2 - 2\mu_1 + 2}{18}$

$= F_m = -\left(\frac{18}{2 - \mu_1}\right)$

$2 = 6 - 3\mu_1$

$3\mu_1 = 4$

$\mu_1 = \frac{4}{3}$

23.(3) $P_1 + P_2 = P$

$$\frac{\lambda}{\lambda_x}-\frac{\lambda}{\lambda_y}=\frac{\lambda}{\lambda}$$

$$\lambda=\frac{\lambda_x\lambda_y}{\lambda_y-\lambda_x}$$

24.(3) $(I-I_g)\times S=I_gG$

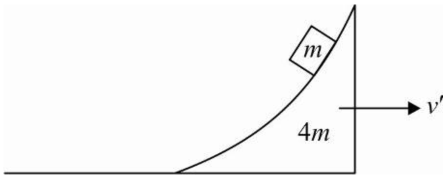
$$S=\left(\frac{0.002\times 50}{0.5-0.002}\right)=0.2\Omega$$

25.(2) The physical size of antenna of receiver and transmitter both inversely proportional to carrier frequency.

26.(1) By conservation of L momentum

$$mv_0=5mv'$$

$$v'=\frac{mv_0}{5m}=\frac{v_0}{5}$$



Conservation of M.E.

$$\frac{1}{2}mv_0^2=\frac{1}{2}5m(v')^2+mgH\ ,\qquad H=\frac{2v_0^2}{5g}$$

27.(4) $\phi_q=\dfrac{\mu_0 i_1 R^2}{2(R^2+x^2)^{3/2}}\times\pi r^2=10^{-3}$

$$\phi_p=\frac{\mu_0 i_1 r^2}{2(\mathrm{r}^2+x^2)^{3/2}}\times\pi R^2$$

$$\frac{\phi_P}{\phi_Q}=\frac{i_2}{i_1}\cdot\frac{(R^2+x^2)^{3/2}}{(r^2+x^2)^{3/2}}=\frac{\phi_P}{10^{-3}}$$

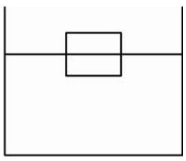
$$\frac{2}{3}=\frac{\phi_P}{10^{-3}}$$

$$\phi_P=6.67\times 10^{-4}$$

28.(4) Initially $\dfrac{4}{5}V\rho\omega g=mg$

After pouring of oil $mg=\rho\omega\dfrac{V}{2}g+\rho_0\dfrac{V}{2}g$

$$\frac{\rho\omega}{2}+\frac{\rho_0}{2}=\frac{4}{5}\rho\omega$$



$$\frac{\rho_0}{2}=\rho\omega\left[\frac{4}{5}-\frac{1}{2}\right]$$

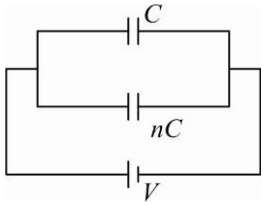
$$\frac{\rho_0}{2}=\rho\omega\left[\frac{8-5}{10}\right]$$

$$\rho_0=\frac{3}{5}\rho\omega$$

29.(1) $Q=C_{eq}V$ Q = Equivalent

$$Q=(n+1)CV$$

After intersection dielectric equivalent C_{eq}



$C'_{eq} = KC + nC$

$C'_{eq} = C(n+K)$

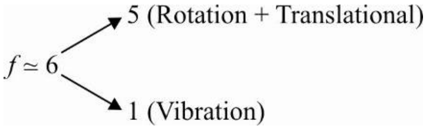
Conservation of charge

$V = \dfrac{(n+1)CV}{n(n+K)C} = \left(\dfrac{n+1}{n+K}\right)V$

30.(4) **For *A***

$R = C_p - C_v = 7$

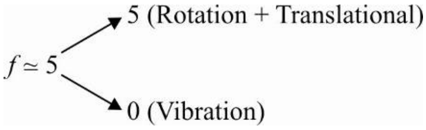
$C_v = \dfrac{fR}{2} = 22 \Rightarrow f = \dfrac{44}{7} = 6.3$



For *B*

$R = C_p - C_v = 9$

$C_v = \dfrac{fR}{2} = 21 \Rightarrow f = \dfrac{42}{9}$



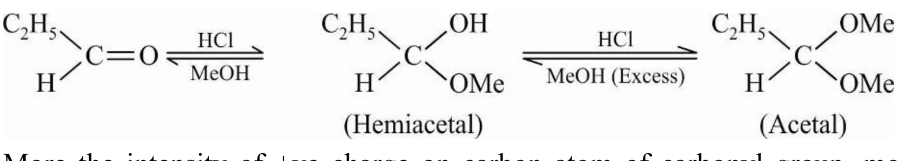
PART-B	CHEMISTRY

1.(2) Kieselghur is an amorphous form of slica.

2.(1) CO is diamagnetic according to molecular orbital theory.

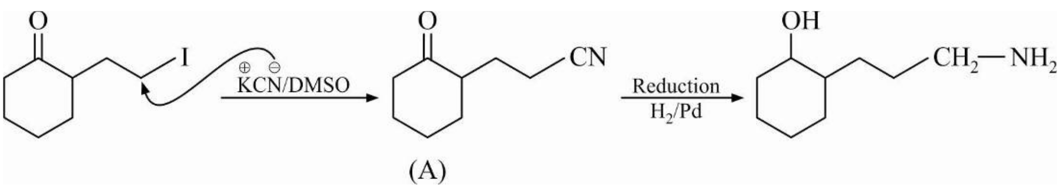
3.(2) By passing 0.1 Faraday electricity, 0.1 gm-equivalents of Ni^{+2} will be discharged.
Number of gm-equivalent = (n – factor) × number of moles
⇒ 0.1 = 2 × number of moles
⇒ Number of moles $= \dfrac{0.1}{2} = 0.05$

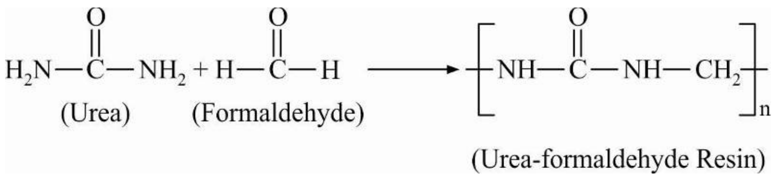
4.(2)



More the intensity of +ve charge on carbon atom of carbonyl group, more is the reactivity towards nucleophilic addition reaction. So propanal is more reactivity than acetone.

5.(2)



6.(3) 

7.(1) Noradrenaline is one of the neurotransmitters that plays a role in mood change

8.(1) Assertion is correct as Haematite ore is used for extraction of Fe.
Haematite is an oxide ore, so reason is incorrect.

9.(1) Neptunium (Np) and plutonium (Pu) show maximum number of oxidation states starting from +3 to +7.

10.(2) 20% (mass/mass) means
100 gm of solution has 20 gm of KI
$\Rightarrow$ 80 gm of solvent contains 20 gm of KI

Molality, $m = \frac{20}{166} \times \frac{1000}{80}$

$= 1.506 \qquad \approx 1.51\,m$

11.(3) According to the postulates of VBT we can't predict the magnetic properties of complexes, it can't distinguish ligands as weak and strong field thus is not able to explain colour of complexes.

12.(1) $z = 1 + \frac{Pb}{RT}$ for inert gases

$z \propto b$, 'b' depends on size of the gas atom the gas atom therefore steepest increase in plot of z is maximum for Xenon.

13.(1) Boron trioxide (B_2O_3) is acidic
Al_2O_3 and Ga_2O_3 amphoteric
In_2O_3 and Tl_2O_3 are basic

14.(2) Initially solution is of NaOH therefore pH is high then it starts decreasing with addition of HCl. After equivalence point pH will become constant and in acidic range due to presence of strong acid.

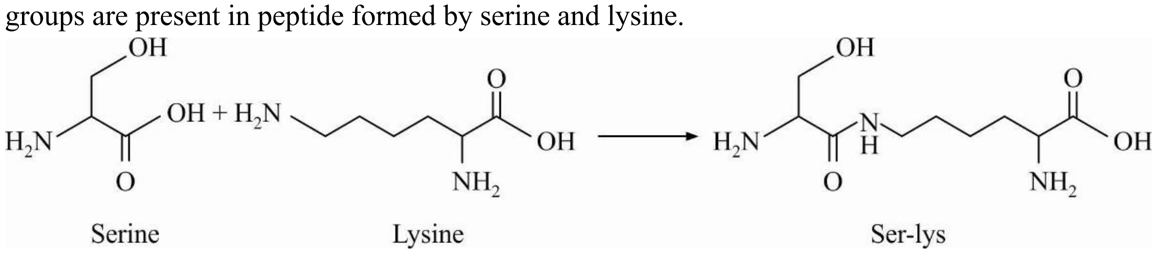
15.(1) Due to intermolecular hydrogen bonding HF has highest b.p. among hydrogen halides.

16.(1) $\Delta U = q + W$
$\Delta U = (-2) + 10 = 8\,kJ$

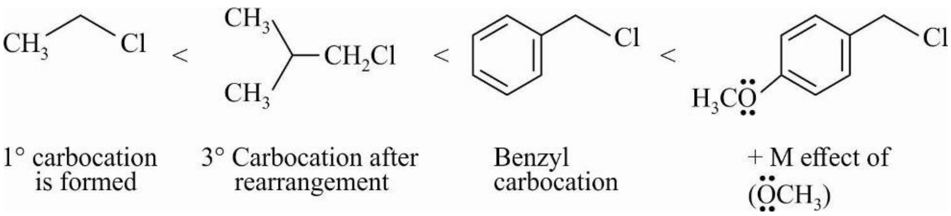
17.(3) As the distance from nucleus increases total energy increases and is minimum at distance a_0.

18.(1) Hinsberg's reagent is benzenesulphonylchloride.

19.(2) Ceric ammonium nitrate test is for alcohols and carbylamines test is for amines. Both these functional groups are present in peptide formed by serine and lysine.



20.(4) More stable is the carbocation formed more is the reactivity for S_N1 reactions.



21.(3) $K_2SO_4 = 2K^+ + SO_4^{2-}$

$i = 3$

$Al_f = i \times K_f \times m$ $= 3 \times 4 \times 0.03 = 0.36$ K

22.(4) According to given graph, activation enthalpy to from C is greater than that to from D.

23.(1) Bauxide $\rightarrow Al_2O_3$, Malachite $\rightarrow CuCO_3.Cu(OH)_2$

Siderite $\rightarrow FeCO_3$, Calamine $\rightarrow ZnCO_3$

24.(3) Stratosphere lies between 10 to 50 km from sea level.

25.(4) Number of particles $= \frac{10^{-3} \times 10}{10^3} N_A$

Total area = Area covered by one particle × number of particle $0.24 = \text{Area} \times \frac{10^{-3} \times 10}{10^3} \times 6 \times 10^{23}$

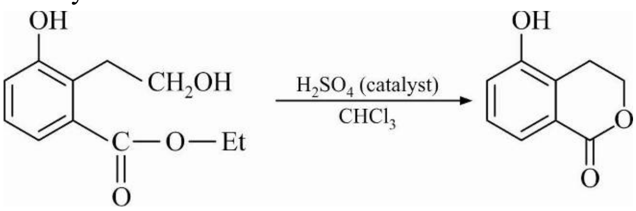
$\text{Area} = 4 \times 10^{-20} \text{cm}^2$

(a) edge length $= 2 \times 10^{-10} \text{cm} = 2 \text{pm}$

26.(1) Common CN of transition elements = 6

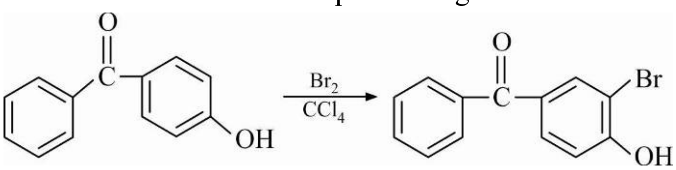
Common CN of inner transition elements = 8 to 12.

27.(1) It is acid catalyzed intermolecular esterification reaction

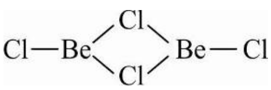


28.(4) S_N1 is a two step reaction in which first step is rate determining step and stable carbocation is formed. So, peak of first step is higher than second step.

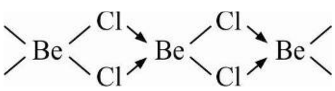
29.(2) Electrophilic substitution is more in phenol ring.



30.(4) In vapour phase, $BeCl_2$ exit as dimer



In solid state, $BeCl_2$ has polymer chain structure



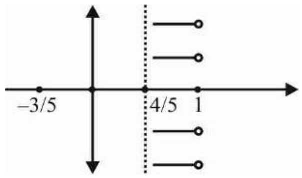
PART-C	MATHEMATICS

1.(4) $w=\frac{5+z}{5(1-z)}$, $\quad 5w-5wz=5+z$

$z=\frac{5w-5}{5w+3}$, $\quad z=\frac{w-1}{w+3/5}$

$|z|=\left|\frac{w-1}{w+3/5}\right|<1$, $\quad |w-1|<\left|w+\frac{3}{5}\right|$

$\text{Re}(w)>\frac{1}{5}$, $\quad 5\text{Re}(w)>1$



2.(4) $x^2+y^2=4$ ... (i)

$x^2+y^2+6x+8y-24=0$

$(x+3)^2+(y+4)^2=49$ ... (ii)

Clearly circle (i) & circle (ii) touches internally because $c_1c_2=|r_1-r_2|$

Equation of common tangent $s_1-s_2=0$

$s_1-s_2=6x+8y-20=0$

$3x+4y=10$

Point (6, –2) passes through the tangent.

3.(1) $I=\int_0^1 x\cdot\cot^{-1}\left(1-x^2+x^4\right)dx$

Put $x^2=t$

$2xdx=dt$

$I=\frac{1}{2}\int_0^1\cot^{-1}\left(1-t+t^2\right)dt$

$=\frac{1}{2}\int_0^1\tan^{-1}\left(\frac{1}{1+t(t-1)}\right)dt$

$=\frac{1}{2}\int_0^1\tan^{-1}\left(\frac{t-(t-1)}{1+t\cdot(t-1)}\right)dt$

$=\frac{1}{2}\int_0^1\left\{\tan^{-1}t+\tan^{-1}(1-t)\right\}dt$

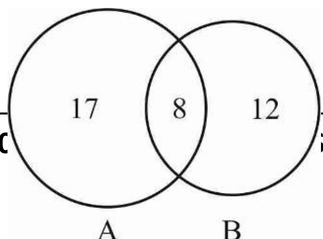
$\therefore \quad \int_0^1\tan^{-1}t\,dt=\int_0^1\tan^{-1}(1-t)\,dt$

$I=\int_0^1\tan^{-1}t\,dt$

$=t\cdot\tan^{-1}t\Big|_0^1-\int_0^1\frac{t}{1+t^2}dt=\frac{\pi}{4}-\frac{1}{2}\ell n\left(1+t^2\right)\Big|_0^1=\frac{\pi}{4}-\frac{1}{2}\ell n2$ Ans.

4.(3) Let number of people in city = 100

$A\cap\overline{B}=17$



30% of $\left(A\cap\bar{B}\right)=5.1$

$\bar{A}\cap B=12$

40% of $\left(\bar{A}\cap B\right)=4.8$

$A\cap B=8$

50% of $\left(A\cap B\right)=4$

Total = 5.1 + 4.8 + 4 = 13.9

5.(4) $x^2\neq 4$ & $x^3-x>0$

$x\neq 2,-2$ ... (i)

$x\cdot(x-1)(x+1)>0$

$x\in(-1,0)\cup(1,\infty)$ ... (ii)

Using (i) & (ii) $x\in(-1,0)\cup(1,2)\cup(2,\infty)$

6.(1) $D=\begin{vmatrix}2 & 3 & -1\\ 1 & k & -2\\ 2 & -1 & 1\end{vmatrix}=0$

$2(k-2)-3\times 5+2k+1=0$

$4k=10$, $k=\dfrac{9}{2}$

Equation (i) $2\cdot\dfrac{x}{y}+3-\dfrac{z}{y}=0$

Equation (ii) $2\cdot\dfrac{x}{y}-1+\dfrac{z}{y}=0$

An adding $\dfrac{x}{y}=\dfrac{-1}{2}$

$\dfrac{z}{y}=2$

$\dfrac{x}{z}=-\dfrac{1}{4}$

$\dfrac{x}{y}+\dfrac{x}{z}+\dfrac{z}{y}+k=\dfrac{1}{2}$

7.(4) $\vec{a}\cdot\hat{i}=\cos\dfrac{\pi}{3}=\dfrac{1}{2}=\ell$

$\vec{a}\cdot\hat{j}=\cos\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}=m$

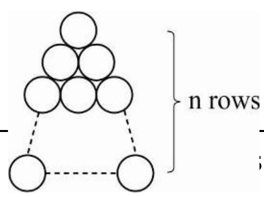
Let $\vec{a}\cdot\hat{k}=n=\cos\theta$

We know that

$\ell^2+m^2+n^2=1$, $\dfrac{1}{4}+\dfrac{1}{2}+n^2=1$

$n=\pm\dfrac{1}{2}=\cos\theta$, $\theta=\dfrac{\pi}{3},\dfrac{2\pi}{3}$

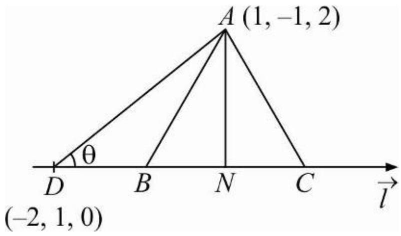
8.(1) $\dfrac{n(n+1)}{2}+99=(n-2)^2$



n = 19

No. Of balls used for the triangle = $19\times\frac{20}{2}$ = 190.

9.(2) $Ar(ABC)=\frac{1}{2}\times BC\times AN$



$AN = AD\sin\theta$

$\overrightarrow{AD}=3\hat{i}-2\hat{j}+2\hat{k}$

$\vec{\ell}=3\hat{i}+4\hat{k}$

$AD\sin\theta=\left|\overrightarrow{AD}\times\hat{l}\right|$

$$=\frac{\left|\left(3\hat{i}-2\hat{j}+2\hat{k}\right)\times\left(3\hat{i}+4\hat{k}\right)\right|}{5}=\frac{\left|-8\hat{i}+6\hat{j}+6\hat{k}\right|}{5}=\frac{\sqrt{136}}{5}$$

$Ar\left(ABC\right)=\frac{1}{2}\times5\times\sqrt{\frac{136}{5}}=\sqrt{34}$.

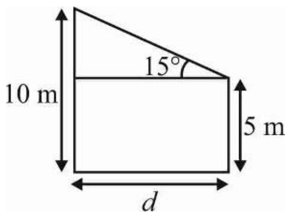
10.(3) $a\left|\pi-5\right|+1=b\left|5-\pi\right|+3$

$a\left(5-\pi\right)+1=b\left(5-\pi\right)+3$

$\left(a-b\right)\left(5-\pi\right)=2$

$a-b=\frac{2}{5-\pi}$

11.(4) $\tan15^\circ=\frac{5}{d}$



$\Rightarrow\quad 2-\sqrt{3}=\frac{5}{d}$

$\Rightarrow\quad d=\frac{5}{2-\sqrt{3}}=5\left(2+\sqrt{3}\right)$.

12.(2) Let a point lying on the parabola $y^2=x$ is $\left(\beta^2,\beta\right)$

Equation of tangent at $\left(\beta^2,\beta\right)$ to the parabola $y^2=x$ is $y\beta=\frac{x+\beta^2}{2}$

$\Rightarrow\quad y=\frac{x}{2\beta}+\frac{\beta}{2}$

If this line also touches the ellipse $\frac{x^2}{1}+\frac{y^2}{1/2}=1$ then $c^2=a^2m^2+b^2$

$\frac{\beta^2}{4}=1\cdot\frac{1}{4\beta^2}+\frac{1}{2}$

$\Rightarrow\quad \beta^4=1+2\beta^2 \qquad \Rightarrow\quad \beta^4-2\beta^2-1=0$

$\left(\beta^2-1\right)^2=2 \qquad \Rightarrow\quad \beta^2-1=\pm\sqrt{2}$

$\beta^2=1\pm\sqrt{2}$ (–ive sign rejected)

$\Rightarrow\quad \beta^2=1+\sqrt{2} \qquad \Rightarrow\quad \alpha=1+\sqrt{2}$

13.(2) $1+2\times3+3\times5+4\times7+....$ upto 11^{th} term

$T_n=n(2n-1)$

$$S_{11}=\sum_{n=1}^{11}T_n=2\sum_{n=1}^{11}n^2-\sum_{n=1}^{11}n=2\times\frac{11\times12\times23}{6}-\frac{11\times12}{2}=946$$

14.(1) $\left(m^2+1\right)x^2-3x+\left(m^2+1\right)^2=0$

Sum of roots $=\dfrac{3}{m^2+1}=\alpha+\beta,\ \alpha\beta=m^2+1$

$\alpha+\beta$ is max. when $m=0$

$\Rightarrow \quad \alpha+\beta=3,\ \alpha\beta=1 \quad \Rightarrow \quad \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=9-2=7$

$$\left|\alpha^3-\beta^3\right|=\left|(\alpha-\beta)\left(\alpha^2+\alpha\beta+\beta^2\right)\right|=\left|\sqrt{(\alpha+\beta)^2-4\alpha\beta}\left(\alpha^2+\alpha\beta+\beta^2\right)\right|$$

$$=\left|\sqrt{9-4}(7+1)\right|=8\sqrt{5}$$

15.(2) $\lim\limits_{x\to2}\displaystyle\int_6^{f(x)}\frac{2t}{x-2}dt$

Given that $f(2)=6$

$$=\lim_{x\to2}\frac{\int_6^{f(x)}2t\,dt}{(x-2)}\quad\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to2}\frac{2f(x)f'(x)-0}{1-0}\quad\text{(Using L.H. Rule)}$$

$$=2\times6f'(2)$$

$$=12f'(2)$$

16.(4) $\int e^{\sec x}\left(\sec x\tan x\,f(x)+\left(\sec x\tan x+\sec^2x\right)\right)dx=e^{\sec x}f(x)+c$

Differentiating both sides w.r.t. x

$\Rightarrow \quad e^{\sec x}\left(\sec x\tan x\ f(x)+\sec x\tan x+\sec^2x\right)=e^{\sec x}f'(x)+f(x)e^{\sec x}\cdot\sec x\tan x$

$\Rightarrow \quad f'(x)=\sec x\tan x+\sec^2x \quad \Rightarrow \quad f(x)=\sec x+\tan x+K$

17.(3) Equation of required plane is $P_1+\lambda P_2=0$

$$2x+3y+z+5+\lambda(x+y+z-6)=0$$

$$\vec{\lambda}=(2+\lambda)\hat{i}+(3+\lambda)\hat{j}+(1+\lambda)\hat{k}$$

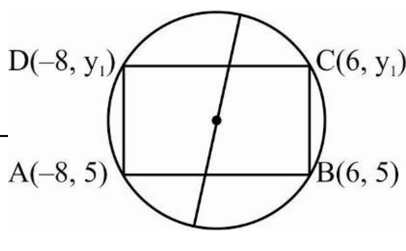
$$\vec{\lambda}\cdot\hat{k}=0$$

$$\lambda=-1$$

Equation of plane $=x+2y+11=0$

Distance $=\dfrac{11}{\sqrt{1^2+2^2}}=\dfrac{11}{\sqrt{5}}$

18.(1) Mid point of $AC\left(-1,\dfrac{5+y_1}{2}\right)$ lies on diameter $3y=x+7$



$\Rightarrow \quad 3\left(\frac{5+y_1}{2}\right)=-1+7$

$\Rightarrow \quad y_1=-1$

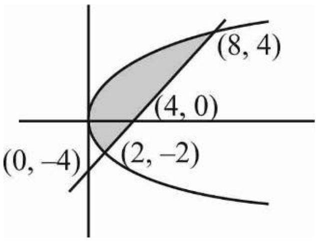
Area of rectangle = 14 × 6 = 84

19.(1) $y^2 \le 2x,\ x \le y+4$

Solve $y^2=2x$ and $x=y+4$; we get (8, 4) & (2, –2)

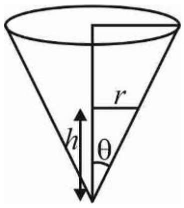
Shaded area $=\int_{-2}^{4}\left((y+4)-\frac{y^2}{2}\right)dy=18$

Ans. (1)



20.(3) $\theta=\tan^{-1}\frac{1}{2}$

$\tan\theta=\frac{1}{2}=\frac{r}{h} \quad \Rightarrow \quad h=2r$



$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi\frac{h^2}{4}\cdot h=\frac{\pi h^3}{12}$

$\frac{dv}{dt}=\frac{\pi}{12}\cdot 3h^2\frac{dh}{dt}$

$\Rightarrow \quad 5=\frac{\pi}{4}\cdot 100\times\frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt}=\frac{1}{5\pi}$

21.(3) $x+(a-1)y=1$

$2x+a^2y=1 \quad a\in R-\{0,1\}$

$m_1m_2=-1$

$-\frac{1}{a-1}\cdot\frac{-2}{a^2}=-1, \quad -2=a^3-a^2$

$a^3-a^2+2=0 \Rightarrow a=-1$

$\therefore$ lines are $x-2y=1$ and $2x+y=1$ $\quad\therefore$ point of intersection of lines is $\left(\frac{3}{5},-\frac{1}{5}\right)$

Distance from origin $=\sqrt{\frac{9}{25}+\frac{1}{25}}=\sqrt{\frac{10}{25}}=\sqrt{\frac{2}{5}}$.

22.(3) $\frac{dy}{dx}-y\tan x=\frac{6x}{\cos x};\ x\in\left(0,\frac{\pi}{2}\right)$

I.F. $=e^{\int -\tan dx}=e^{+\log|\cos x|}=\cos x$

$y\cdot\cos x=\int\frac{6x}{\cos x}\cdot\cos x\,dx, \quad y\cos x=3x^2+c$

$y\left(\frac{\pi}{3},0\right), \quad 0=3\left(\frac{\pi^2}{9}\right)+C$

$C=-\frac{\pi^2}{3}, \quad y\cos x=3x^2-\frac{\pi^2}{3}$

$$y\cos\frac{\pi}{6}=3\left(\frac{\pi^2}{36}\right)-\frac{\pi^2}{3}=\frac{\pi^2}{12}-\frac{\pi^2}{3}=-\frac{3\pi^2}{12}=-\frac{\pi^2}{4}, \qquad y=-\frac{\pi^2}{4}\cdot\frac{2}{\sqrt{3}}=\frac{-\pi^2}{2\sqrt{3}}$$

23.(1) $a-d,\ a,\ a+d \rightarrow$ Terms

$$3a=33 \quad \Rightarrow \quad a=11$$

$$a(a-d)(a+d)=1155$$

$$11\left(121-d^2\right)=1155$$

$$121-d^2=105$$

$$d^2=16 \qquad \Rightarrow \qquad d=4,-4$$

$$T_{11}=15+(10)(-4)=-40+15=-25$$

24.(1) ${}^nC_{r-1}:{}^nC_r:{}^nC_{r+1}=2:15:70$

$${}^nC_{r-1}:{}^nC_r=\frac{2}{15},\ \frac{{}^nC_r}{{}^nC_{r-1}}=\frac{15}{2}$$

$$\frac{n-r+1}{r}=\frac{15}{2}$$

$$2n-2r+2=15r$$

$$2n-17r=-2 \qquad \text{... (i)}$$

$$\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{70}{15}\Rightarrow\frac{n-(r+1)+1}{r+1}=\frac{14}{3}$$

$$\frac{n-r}{r+1}=\frac{14}{3}$$

$\Rightarrow \quad 3n-3r=14r+14$

$$3n-17r=14 \qquad ...(ii)$$
$$2n-17r=-2 \qquad ...(i)$$
$$-\quad +\quad +$$
$$n=16$$

$$48-17r=14$$

$$17r=34$$

$\Rightarrow \quad r=2.$

$$\text{Average }=\frac{{}^{16}C_1+{}^{16}C_2+{}^{16}C_3}{3}=\frac{16+{}^{17}C_3}{3}=\frac{16+\frac{17.16.15}{1.2.3}}{3}=\frac{16+680}{3}=\frac{696}{3}=232$$

25.(3)

$$y^2=4x$$

$$2yy'=4$$

$$y'=\frac{2}{y}$$

Slope of normal $=\frac{-2}{2}=-1$

Slope of normal at P = slope of CP

$$-1=\frac{\beta-2}{\alpha-1}$$

$$-\alpha+1=\beta-2$$

$$\beta=3-\alpha$$

$$(\alpha-1)^2+(\beta-2)^2=\beta^2$$

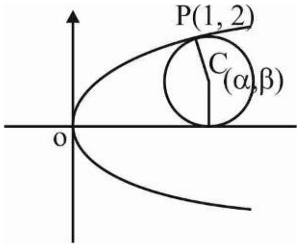
$$\alpha^2+\beta^2-2\alpha-4\beta+5=\beta^2$$

$\alpha^2 - 2\alpha - 4\beta + 5 = 0$

$\alpha^2 - 2\alpha - 4\beta + 5 = 0$ ... (i)

$\alpha^2 - 2\alpha - 12 + 4\alpha + 5 = 0$

$\alpha^2 + 2\alpha - 7 = 0$



$(\alpha + 1)^2 = 8$

$\alpha + 1 = \pm 2\sqrt{2}$

$\alpha = -1 - 2\sqrt{2}, -1 + 2\sqrt{2}$

$\therefore \quad \beta = 3 - \left(-1 + 2\sqrt{2}\right)$

Radius $\beta = 4 - 2\sqrt{2}$

Area $= \pi\left(\beta^2\right)$

$= \pi\left(4 - 2\sqrt{2}\right)^2 = 4\pi\left(2 - \sqrt{2}\right)^2 = 4\pi\left(4 + 2 - 4\sqrt{2}\right) = 8\pi\left(3 - 2\sqrt{2}\right)$

26.(1) $p \Rightarrow (q \cup r)$

Verify with options

$p = T;\ q = F;\ r = F$

$T \Rightarrow (F \cup F)$

$T \Rightarrow F$

27.(4) $\sin 10° \sin 30° \sin 50° \sin 70°$

$\cos 80° \left(\frac{1}{2}\right) \cos 40° \cos 20°$

$\frac{1}{2} \cos 20° \cos 40° \cos 80°$

$\frac{1}{2} \cdot \frac{\sin\left(2^3 \cdot 20°\right)}{2^3 \sin 20°}$

$\frac{1}{16} \frac{\sin 160°}{\sin 20°} = \frac{1}{16} \frac{\sin\left(180° - 20°\right)}{\sin 20°} = \frac{\sin 20°}{16 \sin 20°} = \frac{1}{16}$

28.(2) 10, 22, 26, 29, 34, x, 42, 67, 70, y

$\text{Mean} = \frac{10 + 22 + 26 + 29 + 34 + x + 42 + 67 + 70 + y}{10}$

$42 = \frac{300 + x + y}{10} \quad \Rightarrow \quad 420 = 300 + x + y$

$x + y = 120$ ... (i)

$\text{Median} = \frac{x + 34}{2} = 35 \quad x = 36 \quad y = 84 \quad \therefore \quad \frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

29.(1) $A^T A = 3I_3$

$$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2R & y & -1 \\ 2x & -y & +1 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$8x^2 = 3$

$x = \pm\sqrt{\frac{3}{8}}$; $6y^2 = 3 \Rightarrow y = \pm\sqrt{\frac{1}{2}}$

Ans. 4

30.(4) $F(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x \to 4^+} F(x) = \lim_{h \to 0} F(4+h) = \lim_{h \to 0}\left([4+h] - \left[\frac{4+h}{4}\right]\right) = \lim_{h \to 0}(4-1) = 3$$

$$\lim_{x \to 4^-} F(x) = \lim_{h \to 0} F(4-h) = \lim_{h \to 0}\left([4-h] - \left[\frac{4-h}{4}\right]\right) = \lim_{h \to 0}(3-0) = 3$$

$F(4) = [4] - \left[\frac{4}{4}\right] = 4 - 1 = 3$

$F(x)$ is continuous at $x = 4$.